January - 2017

# MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE (COMPUTER SCIENCE \& ENGINEERING) 

Duration: 3 hours<br>Max. Marks: 70

Answer any FIVE questions. All questions carry equal marks

1. a) Prove that for any propositions $\mathrm{p}, \mathrm{q}$ and r , the compound proposition $\{\mathrm{P} \rightarrow(\mathrm{Q} \rightarrow \mathrm{R})\} \rightarrow\{(\mathrm{P} \rightarrow \mathrm{Q}) \rightarrow(\mathrm{P} \rightarrow \mathrm{R})\}$ is a tautology.
b) Construct a truth table for each of these following statements.
i) $\left(\mathrm{A}^{\wedge} \sim \mathrm{B}\right) \rightarrow(\mathrm{C} \vee \mathrm{D})$
ii) $(\mathrm{P} \vee \mathrm{Q}) \oplus(\mathrm{P} \wedge \mathrm{Q})$
2. a) Show that $r \rightarrow s$ can be derived from

$$
\mathrm{p} \rightarrow(\mathrm{q} \rightarrow \mathrm{~s}), \mathrm{r} \vee \mathrm{p} \text { and } \mathrm{q} .
$$

b) (i) Derive Predicate, Quantifiers and (ii) Write the rules of inferences.
3. a) Draw the hasse diagram representing the positive divisors of 36 .
b) Verify that $\mathrm{R}=\{(1,1),(2,2),(3,3),(4,4),(1,2),(2,1)\}$ is an equivalence relation on the set $\mathrm{A}=\{1,2,3,4\}$ find the corresponding partition on A .
4. a) Prove that $\left\langle Z_{5}^{*},.\right\rangle$ is a cyclic group. Find all its generators
b) Given $\mathrm{A}=\{2,3,4\}, \mathrm{B}=\{1,2\}$ and $\mathrm{C}=\{4,5,6\}$ find $\mathrm{A}+\mathrm{B}$, $\mathrm{B}+\mathrm{C}, \mathrm{A}+\mathrm{B}+\mathrm{C}$ and $(\mathrm{A}+\mathrm{B})+(\mathrm{B}+\mathrm{C})$
5. a) Define Pigeon hole principle

7 M
b) How many solutions are there to the equation $x_{1}+x_{2}+x_{3}+x_{4}+x_{5}+x_{6}=29$, where $x_{i}, i=1,2,3,4,5,6$ is a nonnegative integer such that
i) $x_{i}>1$ for $i=1,2,3,4,5,6$ ?
ii) $\mathrm{x}_{1}<=5$ ?
iii) $\mathrm{x}_{1}<8$ and $\mathrm{x}_{2}>8$ ?
6. a) In a survey of 60 people it was found that 25 read weekly magazines, 26 read fortnightly magazines, 26 read monthly magazines, 9 read both weekly and monthly magazines, 11 read both weekly and fortnightly magazines, 8 read both fortnightly and monthly magazines and 3 read all three magazines. Find
i) The number of people who read at least one of the three magazines, and
ii) The number of people who read exactly one magazine
b) Solve the recurrence relation $a_{n}=8 a_{n-1}+10^{n-1}$ with $a_{1}=1$ using generating function method.
7. a) Find the shortest path from a to z in the graph shown below using Dijkstra's algorithm.

b) Write short notes on 7 M
i) Graph isomorphism
ii) Planar graphs
iii) Graph coloring
8. a) Write an algorithm for Breadth- first search.
b) Find minimum spanning tree for the following graph using Kruskal's algorithm.


